

Two-photon spin states and entanglement states

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Abstract

In this paper, we have given the spin states of two-photon, which are expressed by the quadratic combination of two single photon spin states, they are a kind of quantum expression. Otherwise, we give all entanglement states of two-photon, which are different from the xanzsd classical polarization vector expression of two-photon entanglement state.

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1 Introduction

Quantum entanglement, and its inherent non local properties, are among the most fascinating and challenging features of the quantum world. In addition, entanglement plays a central role in quantum information [1–5]. Since its first description in the decade of 1930 [6], and in spite of the decisive contribution of Bell [7] and the subsequent experimental studies [8], entanglement stays even now as a rather mysterious and puzzling property of bipartite quantum objects. Entanglement is not only a fundamental concept in Quantum Mechanics with profound implications, but also a basic ingredient of many recent technological applications that has been put forward in quantum communications and quantum computing [9, 10]. Entanglement is a very special type of correlation between particles that can exist in spite of how distant they are. Nevertheless, the term entanglement is sometimes also used to refer to certain correlations existing between different degrees of freedom of a single particle [11]. By and large, the most common method to generate photonic entanglement, that is entanglement between photons, is the process of spontaneous parametric

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down-conversion (SPDC) [12]. In SPDC, two lower-frequency photons are generated when an intense higher-frequency pump beam interacts with the atoms of a non-centrosymmetric nonlinear crystal. Entanglement can reside in any of the degrees of freedoms that characterize light: angular momentum (polarization and orbital angular momentum), momentum and frequency, or in several of them, what is known as hyper-entanglement. Undoubtedly, polarization is the most widely used resource to generate entanglement between photons thanks to the existence of many optical elements to control the polarization of light and to the easiness of its manipulation when compared to other characteristics of a light beam, e.g., its spatial shape or bandwidth.

Entangled states of photons are the basic resource in the successful implementation of quantum information processing applications, namely optical quantum computing [13, 14], and quantum cryptography, or quantum key distribution [15, 16]. Also, in the experimental study of fundamental problems in Quantum Mechanics, as loophole free tests of the violation of the Bell's inequalities [17], the delayed-choice quantum eraser [18], quantum teleportation and entanglement swapping [19], generation of states with a large number of particles and generalized types of entanglement [20], etc. In this paper, we have given the spin states of two-photon, which are expressed by the quadratic combination of two single photon spin states, they are a kind of quantum expression. Otherwise, we give all entanglement states of two-photon, which are different from the xanzsd classical polarization vector expression of two-photon entanglement state.

2 The spin state of two-photon

The following will give some comments on the real meaning of the state of photon. In quantum electrodynamics, the spin vector operator of the photon is [21, 22] (in natural unit system $\hbar = c = 1$)

$$s_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, s_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, s_z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (1)$$

$$\vec{S}^2 = s_x^2 + s_y^2 + s_z^2 = 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (2)$$

the spin vector χ_μ of S^2 and s_z satisfy the following equations

$$\vec{S}^2 \chi_\mu = 2 \chi_\mu, \quad (3)$$

$$s_z \chi_\mu = \mu \chi_\mu, \quad (4)$$

these are

$$\chi_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \chi_1 = -\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}, \chi_{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}, \quad (5)$$

the total spin \vec{S} is the sum of two photons spin s_1 and s_2

$$\vec{S} = \vec{s}_1 + \vec{s}_2, \quad (6)$$

and \vec{S}^2 eigenvalue is

$$\vec{S}^2 = S(S+1), \quad (7)$$

and quantum number S are

$$S = 0, 1, 2. \quad (8)$$

For the given S , s_z eigenvalues are: $\mu = -S, -S+1, \dots, S$, i.e., there are $2S+1$ eigenvalues, and $2S+1$ eigenfunctions of spin. For $S=0$, there is one eigenfunction, for $S=1$, there are three eigenfunctions, and for $S=2$, there are five eigenfunctions, i.e., there are nine spin wave-functions $\chi_{S\mu}$ for two-photon, which are expressed by the quadratic combination of two single photon spin wave functions $\chi_{\mu 1}$ and $\chi_{\mu 2}$. We try to write the nine spin wave functions of two-photon, they are

(1) $S=2$ spin wave functions

$$\chi_S^{(1)} = \chi_{22}(s_{1z}, s_{2z}) = \chi_1(s_{1z})\chi_1(s_{2z}), \quad (S=2, \mu=2) \quad (9)$$

$$\chi_S^{(2)} = \chi_{21}(s_{1z}, s_{2z}) = \frac{1}{\sqrt{2}}[\chi_0(s_{1z})\chi_1(s_{2z}) + \chi_0(s_{2z})\chi_1(s_{1z})], \quad (S=2, \mu=1) \quad (10)$$

$$\chi_S^{(3)} = \chi_{20}(s_{1z}, s_{2z}) = \frac{1}{\sqrt{2}}[\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z})], \quad (S=2, \mu=0) \quad (11)$$

$$\chi_S^{(4)} = \chi_{2-1}(s_{1z}, s_{2z}) = \frac{1}{\sqrt{2}}[\chi_0(s_{1z})\chi_{-1}(s_{2z}) + \chi_0(s_{2z})\chi_{-1}(s_{1z})], \quad (S=2, \mu=-1) \quad (12)$$

$$\chi_S^{(5)} = \chi_{2-2}(s_{1z}, s_{2z}) = \chi_{-1}(s_{1z})\chi_{-1}(s_{2z}), \quad (S=2, \mu=-2) \quad (13)$$

(2) $S=0$ spin wave function

$$\chi_S^{(6)} = \chi_{00}(s_{1z}, s_{2z}) = \chi_0(s_{1z})\chi_0(s_{2z}), \quad (S=0, \mu=0) \quad (14)$$

(3) $S=1$ spin wave functions

$$\chi_A^{(1)} = \chi_{11}(s_{1z}, s_{2z}) = \frac{1}{\sqrt{2}}[\chi_0(s_{1z})\chi_1(s_{2z}) - \chi_0(s_{2z})\chi_1(s_{1z})], \quad (S=1, \mu=1) \quad (15)$$

$$\chi_A^{(2)} = \chi_{10}(s_{1z}, s_{2z}) = \frac{1}{\sqrt{2}}[\chi_1(s_{1z})\chi_{-1}(s_{2z}) - \chi_1(s_{2z})\chi_{-1}(s_{1z})], \quad (S=1, \mu=0) \quad (16)$$

$$\chi_A^{(3)} = \chi_{1-1}(s_{1z}, s_{2z}) = \frac{1}{\sqrt{2}}[\chi_0(s_{1z})\chi_{-1}(s_{2z}) - \chi_0(s_{2z})\chi_{-1}(s_{1z})], \quad (S = 1, \mu = -1) \quad (17)$$

In the following, we shall check the above spin wave functions, the two-photon total spin square \vec{S}^2 and its z component s_z are

$$\begin{aligned} \vec{S}^2 &= (\vec{s}_1 + \vec{s}_2)^2 = \vec{s}_1^2 + \vec{s}_2^2 + 2(s_{1x}s_{2x} + s_{1y}s_{2y} + s_{1z}s_{2z}) \\ &= 4 + 2(s_{1x}s_{2x} + s_{1y}s_{2y} + s_{1z}s_{2z}), \end{aligned} \quad (18)$$

$$s_z = s_{1z} + s_{2z}, \quad (19)$$

by spin wave functions (5), we have

$$s_x\chi_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}(\chi_1 + \chi_{-1}), \quad (20)$$

$$s_x\chi_1 = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\chi_0, \quad (21)$$

$$s_x\chi_{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\chi_0, \quad (22)$$

$$s_y\chi_0 = -\frac{i}{\sqrt{2}}(\chi_1 - \chi_{-1}), \quad (23)$$

$$s_y\chi_1 = \frac{i}{\sqrt{2}}\chi_0, \quad (24)$$

$$s_y\chi_{-1} = -\frac{i}{\sqrt{2}}\chi_0, \quad (25)$$

$$s_z\chi_0 = 0 \cdot \chi_0, \quad (26)$$

$$s_z\chi_1 = 1 \cdot \chi_1, \quad (27)$$

$$s_z\chi_{-1} = -1 \cdot \chi_{-1}, \quad (28)$$

with equations (20)-(28), we can check spin wave functions.

(1) checking $S = 2$ spin wave functions

(a) checking spin wave function $\chi_S^{(1)}$:

$$\begin{aligned} s_z\chi_S^{(1)} &= (s_{1z} + s_{2z})\chi_1(s_{1z})\chi_1(s_{2z}) \\ &= (s_{1z}\chi_1(s_{1z}))\chi_1(s_{2z}) + \chi_1(s_{1z})(s_{2z}\chi_1(s_{2z})) \\ &= 2\chi_1(s_{1z})\chi_1(s_{2z}), \end{aligned} \quad (29)$$

i.e.,

$$\mu = 2, \quad (30)$$

$$\begin{aligned} \vec{S}^2 \chi_S^{(1)} &= [4 + 2(s_{1x}s_{2x} + s_{1y}s_{2y} + s_{1z}s_{2z})] \chi_1(s_{1z}) \chi_1(s_{2z}) \\ &= 4\chi_1(s_{1z}) \chi_1(s_{2z}) + 2\left[\frac{1}{\sqrt{2}}\chi_0(s_{1z})\frac{1}{\sqrt{2}}\chi_0(s_{2z}) + \frac{i}{\sqrt{2}}\chi_0(s_{1z})\frac{i}{\sqrt{2}}\chi_0(s_{2z}) + \chi_1(s_{1z})\chi_1(s_{2z})\right] \\ &= 6\chi_1(s_{1z})\chi_1(s_{2z}), \end{aligned} \quad (31)$$

i.e.,

$$S = 2, \quad (32)$$

we have checked $\chi_S^{(1)}$ is the spin wave function of two-photon corresponding to $S = 2$ and $\mu = 2$.

(b) checking spin wave function $\chi_S^{(2)}$:

$$\begin{aligned} s_z \chi_S^{(2)} &= (s_{1z} + s_{2z}) \frac{1}{\sqrt{2}} [\chi_0(s_{1z})\chi_1(s_{2z}) + \chi_0(s_{2z})\chi_1(s_{1z})] \\ &= \frac{1}{\sqrt{2}} [\chi_0(s_{2z})\chi_1(s_{1z}) + \chi_0(s_{1z})\chi_1(s_{2z})], \end{aligned} \quad (33)$$

i.e.,

$$\mu = 1, \quad (34)$$

$$\begin{aligned} \vec{S}^2 \chi_S^{(2)} &= [4 + 2(s_{1x}s_{2x} + s_{1y}s_{2y} + s_{1z}s_{2z})] \frac{1}{\sqrt{2}} [\chi_0(s_{1z})\chi_1(s_{2z}) + \chi_0(s_{2z})\chi_1(s_{1z})] \\ &= 2\sqrt{2} [\chi_0(s_{1z})\chi_1(s_{2z}) + \chi_0(s_{2z})\chi_1(s_{1z})] \\ &\quad + \sqrt{2} \left[\frac{1}{2} (\chi_1(s_{1z})\chi_0(s_{2z}) + \chi_{-1}(s_{1z})\chi_0(s_{2z})) + \frac{1}{2} (\chi_1(s_{2z})\chi_0(s_{1z}) + \chi_{-1}(s_{2z})\chi_0(s_{1z})) \right] \\ &\quad + \sqrt{2} \left[\frac{1}{2} (\chi_1(s_{1z})\chi_0(s_{2z}) - \chi_{-1}(s_{1z})\chi_0(s_{2z})) + \frac{1}{2} (\chi_1(s_{2z})\chi_0(s_{1z}) - \chi_{-1}(s_{2z})\chi_0(s_{1z})) \right] \\ &= 6 \cdot \frac{1}{\sqrt{2}} (\chi_0(s_{1z})\chi_1(s_{2z}) + \chi_0(s_{2z})\chi_1(s_{1z})), \end{aligned} \quad (35)$$

i.e.,

$$S = 2, \quad (36)$$

we have checked $\chi_S^{(2)}$ is the spin wave function of two-photon corresponding to $S = 2$ and $\mu = 1$.

(c) checking spin wave function $\chi_S^{(3)}$:

$$\begin{aligned}
s_z \chi_S^{(3)} &= (s_{1z} + s_{2z}) \frac{1}{\sqrt{2}} [\chi_1(s_{1z}) \chi_{-1}(s_{2z}) + \chi_1(s_{2z}) \chi_{-1}(s_{1z})] \\
&= \frac{1}{\sqrt{2}} [(s_{1z} \chi_1(s_{1z})) \chi_{-1}(s_{2z}) + \chi_1(s_{2z}) (s_{1z} \chi_{-1}(s_{1z}))] \\
&\quad + \frac{1}{\sqrt{2}} [\chi_1(s_{1z}) (s_{2z} \chi_{-1}(s_{2z})) + (s_{2z} \chi_1(s_{2z})) \chi_{-1}(s_{1z})] \\
&= 0,
\end{aligned} \tag{37}$$

i.e.,

$$\mu = 0, \tag{38}$$

$$\begin{aligned}
\vec{S}^2 \chi_S^{(3)} &= [4 + 2(s_{1x}s_{2x} + s_{1y}s_{2y} + s_{1z}s_{2z})] \frac{1}{\sqrt{2}} [\chi_1(s_{1z}) \chi_{-1}(s_{2z}) + \chi_1(s_{2z}) \chi_{-1}(s_{1z})] \\
&= 2\sqrt{2} [\chi_1(s_{1z}) \chi_{-1}(s_{2z}) + \chi_1(s_{2z}) \chi_{-1}(s_{1z})] \\
&\quad + \sqrt{2} \left[\frac{1}{\sqrt{2}} \chi_0(s_{1z}) \frac{1}{\sqrt{2}} \chi_0(s_{1z}) + \frac{1}{\sqrt{2}} \chi_0(s_{2z}) \frac{1}{\sqrt{2}} \chi_0(s_{1z}) \right] \\
&\quad + \sqrt{2} \left[\frac{i}{\sqrt{2}} \chi_0(s_{1z}) \left(-\frac{1}{\sqrt{2}}\right) \chi_0(s_{2z}) + \frac{i}{\sqrt{2}} \chi_0(s_{2z}) \left(-\frac{1}{\sqrt{2}}\right) \chi_0(s_{1z}) \right] \\
&\quad + \sqrt{2} [-\chi_1(s_{1z}) \chi_{-1}(s_{2z}) - \chi_1(s_{2z}) \chi_{-1}(s_{1z})] \\
&= 2 \cdot \frac{1}{\sqrt{2}} [\chi_1(s_{1z}) \chi_{-1}(s_{2z}) + \chi_1(s_{2z}) \chi_{-1}(s_{1z})] \\
&\quad + 2 \cdot \frac{1}{\sqrt{2}} [\chi_0(s_{1z}) \chi_0(s_{2z}) + \chi_0(s_{2z}) \chi_0(s_{1z})],
\end{aligned} \tag{39}$$

we find that $\chi_S^{(3)}$ is not the common eigenstate of $\{\vec{S}^2, s_z\}$, corresponding to $S = 2$ and $\mu = 0$. Their common eigenstate $\chi_S^{(3)}$ can be obtained by the linear superposition of χ_{20} and χ_{00} , the $\chi_S^{(3)}$ can be written as

$$\chi_S^{(3)} = a \chi_0(s_{1z}) \chi_0(s_{2z}) + b [\chi_1(s_{1z}) \chi_{-1}(s_{2z}) + \chi_1(s_{2z}) \chi_{-1}(s_{1z})], \tag{40}$$

by the eigenequations

$$\begin{cases} \vec{S}^2 \chi_S^{(3)} = 6 \chi_S^{(3)} \\ s_z \chi_S^{(3)} = 0 \end{cases}, \tag{41}$$

we can calculate the superposition coefficients a and b

$$\begin{aligned}
s_z \chi_S^{(3)} &= a (s_{1z} \chi_0(s_{1z})) \chi_0(s_{2z}) + b [(s_{1z} \chi_1(s_{1z})) \chi_{-1}(s_{2z}) + (s_{1z} \chi_{-1}(s_{1z})) \chi_1(s_{2z})] \\
&\quad + a (s_{2z} \chi_0(s_{2z})) \chi_0(s_{1z}) + b [(s_{2z} \chi_{-1}(s_{2z})) \chi_1(s_{1z}) + (s_{2z} \chi_1(s_{2z})) \chi_{-1}(s_{1z})] \\
&= b [\chi_1(s_{1z}) \chi_{-1}(s_{2z}) - \chi_1(s_{2z}) \chi_{-1}(s_{1z})] + b [-\chi_1(s_{1z}) \chi_{-1}(s_{2z}) + \chi_1(s_{2z}) \chi_{-1}(s_{1z})] \\
&= 0,
\end{aligned} \tag{42}$$

i.e.,

$$\mu \equiv 0, \quad (43)$$

$$\begin{aligned}
\vec{S}^2 \chi_S^{(3)} &= 4[a\chi_0(s_{1z})\chi_0(s_{2z}) + b(\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z}))] \\
&\quad + 2[a(s_{1x}\chi_0(s_{1z}))(s_{2x}\chi_0(s_{2z})) + b((s_{1x}\chi_1(s_{1z}))(s_{2x}\chi_{-1}(s_{2z})) + (s_{1x}\chi_{-1}(s_{1z}))(s_{2x}\chi_1(s_{2z})))] \\
&\quad + 2[a(s_{1y}\chi_0(s_{1z}))(s_{2y}\chi_0(s_{2z})) + b((s_{1y}\chi_1(s_{1z}))(s_{2y}\chi_{-1}(s_{2z})) + (s_{1y}\chi_{-1}(s_{1z}))(s_{2y}\chi_1(s_{2z})))] \\
&\quad + 2[a(s_{1z}\chi_0(s_{1z}))(s_{2z}\chi_0(s_{2z})) + b((s_{1z}\chi_1(s_{1z}))(s_{2z}\chi_{-1}(s_{2z})) + (s_{1z}\chi_{-1}(s_{1z}))(s_{2z}\chi_1(s_{2z})))] \\
&\quad + 2[b(-\chi_1(s_{1z}))\chi_{-1}(s_{2z}) - (\chi_{-1}(s_{1z}))\chi_1(s_{2z})] \\
&= 4[a\chi_0(s_{1z})\chi_0(s_{2z}) + b(\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z}))] + 2[a(\chi_1(s_{1z})\chi_{-1}(s_{2z}) \\
&\quad + \chi_1(s_{2z})\chi_{-1}(s_{1z})) + 2b\chi_0(s_{1z})\chi_0(s_{2z}) - b(\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z}))] \\
&= 4(a+b)\chi_0(s_{1z})\chi_0(s_{2z}) + 2(a+b)(\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z})) \\
&= 6[a\chi_0(s_{1z})\chi_0(s_{2z}) + b(\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z}))], \quad (44)
\end{aligned}$$

comparing the coefficients of the same terms, we obtain

$$\begin{cases} 4(a+b) = 6a \\ 2(a+b) = 6b \end{cases}, \quad (45)$$

i.e.,

$$a = 2b, \quad (46)$$

then

$$\chi_S^{(3)} = 2b\chi_0(s_{1z})\chi_0(s_{2z}) + b[\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z})], \quad (47)$$

by condition of normalization

$$(\chi_S^{(3)})^+ \chi_S^{(3)} = 1, \quad (48)$$

we have

$$b = \frac{1}{\sqrt{6}}, \quad (49)$$

we get the eigenstate $\chi_S^{(3)}$, it is

$$\chi_S^{(3)} = \frac{2}{\sqrt{6}}\chi_0(s_{1z})\chi_0(s_{2z}) + \frac{1}{\sqrt{6}}[\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z})], \quad (50)$$

(d) checking spin wave function $\chi_S^{(4)}$:

$$\begin{aligned}
s_z \chi_S^{(4)} &= \frac{1}{\sqrt{2}} [(s_{1z} \chi_0(s_{1z})) \chi_{-1}(s_{2z}) + \chi_0(s_{2z}) (s_{1z} \chi_{-1}(s_{1z}))] \\
&\quad + \frac{1}{\sqrt{2}} [\chi_0(s_{1z}) (s_{2z} \chi_{-1}(s_{2z})) + (s_{2z} \chi_0(s_{2z})) \chi_{-1}(s_{1z})] \\
&= -\frac{1}{\sqrt{2}} [\chi_0(s_{1z}) \chi_{-1}(s_{2z}) + \chi_0(s_{2z}) \chi_{-1}(s_{1z})],
\end{aligned} \tag{51}$$

i.e.,

$$\mu = -1, \tag{52}$$

$$\begin{aligned}
\vec{S}^2 \chi_S^{(4)} &= 4\chi_S^{(4)} + 2\frac{1}{\sqrt{2}} [(s_{1x} \chi_0(s_{1z})) (s_{2x} \chi_{-1}(s_{2z})) + (s_{2x} \chi_0(s_{2z})) (s_{1x} \chi_{-1}(s_{1z}))] \\
&\quad + 2\frac{1}{\sqrt{2}} [(s_{1y} \chi_0(s_{1z})) (s_{2y} \chi_{-1}(s_{2z})) + (s_{2y} \chi_0(s_{2z})) (s_{1y} \chi_{-1}(s_{1z}))] \\
&\quad + 2\frac{1}{\sqrt{2}} [(s_{1z} \chi_0(s_{1z})) (s_{2z} \chi_{-1}(s_{2z})) + (s_{2z} \chi_0(s_{2z})) (s_{1z} \chi_{-1}(s_{1z}))] \\
&= 4\chi_S^{(4)} + 2\frac{1}{\sqrt{2}} [\chi_{-1}(s_{1z}) \chi_0(s_{2z}) + \chi_{-1}(s_{2z}) \chi_0(s_{1z})] \\
&= 6\chi_S^{(4)},
\end{aligned} \tag{53}$$

i.e.,

$$S = 2, \tag{54}$$

we have checked $\chi_S^{(4)}$ is the spin wave function of two-photon corresponding to $S = 2$ and $\mu = -1$.

(e) checking spin wave function $\chi_S^{(5)}$:

$$\begin{aligned}
s_z \chi_S^{(5)} &= (s_{1z} \chi_{-1}(s_{1z})) \chi_{-1}(s_{2z}) + \chi_{-1}(s_{1z}) (s_{2z} \chi_{-1}(s_{2z})) \\
&= -\chi_1(s_{1z}) \chi_{-1}(s_{2z}) - \chi_{-1}(s_{1z}) \chi_1(s_{2z}) \\
&= -2[\chi_1(s_{1z}) \chi_{-1}(s_{2z})],
\end{aligned} \tag{55}$$

i.e.,

$$\mu = -2, \tag{56}$$

$$\begin{aligned}
\vec{S}^2 \chi_S^{(5)} &= 4\chi_{-1}(s_{1z}) \chi_{-1}(s_{2z}) + 2[(s_{1x} \chi_{-1}(s_{1z})) (s_{2x} \chi_{-1}(s_{2z})) \\
&\quad + (s_{1y} \chi_{-1}(s_{1z})) (s_{2y} \chi_{-1}(s_{2z})) + (s_{1z} \chi_{-1}(s_{1z})) (s_{2z} \chi_{-1}(s_{2z}))] \\
&= 4\chi_{-1}(s_{1z}) \chi_{-1}(s_{2z}) + 2[\frac{1}{\sqrt{2}} \chi_0(s_{1z}) \frac{1}{\sqrt{2}} \chi_0(s_{2z}) + \frac{-i}{\sqrt{2}} \chi_0(s_{1z}) \frac{-i}{\sqrt{2}} \chi_0(s_{2z}) \\
&\quad + \chi_{-1}(s_{1z}) \chi_{-1}(s_{2z})] \\
&= 6\chi_{-1}(s_{1z}) \chi_{-1}(s_{2z}),
\end{aligned} \tag{57}$$

i.e.,

$$S = 2, \quad (58)$$

we have checked $\chi_S^{(5)}$ is the spin wave function of two-photon corresponding to $S = 2$ and $\mu = -2$.

(2) checking $S = 0$ spin wave functions $\chi_S^{(6)}$:

$$s_z \chi_s^{(6)} = (s_{1z} \chi_0(s_{1z})) \chi_0(s_{2z}) + \chi_0(s_{1z}) (s_{2z} \chi_0(s_{2z})) = 0, \quad (59)$$

i.e.,

$$\mu = 0, \quad (60)$$

$$\begin{aligned} \vec{S}^2 \chi_S^{(6)} &= 4\chi_0(s_{1z})\chi_0(s_{2z}) + 2[(s_{1x}\chi_0(s_{1z}))(s_{2x}\chi_0(s_{2z})) \\ &\quad + (s_{1y}\chi_0(s_{1z}))(s_{2y}\chi_0(s_{2z})) + (s_{1z}\chi_0(s_{1z}))(s_{2z}\chi_0(s_{2z}))] \\ &= 4\chi_0(s_{1z})\chi_0(s_{2z}) + 2\left[\frac{1}{\sqrt{2}}(\chi_1(s_{1z}) + \chi_{-1}(s_{1z}))\frac{1}{\sqrt{2}}(\chi_1(s_{2z}) + \chi_{-1}(s_{2z}))\right. \\ &\quad \left.+ \frac{-i}{\sqrt{2}}(\chi_1(s_{1z}) - \chi_{-1}(s_{1z}))\frac{-i}{\sqrt{2}}(\chi_1(s_{2z}) - \chi_{-1}(s_{2z}))\right] \\ &= 4\chi_0(s_{1z})\chi_0(s_{2z}) + 2[\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z})] \neq 0, \end{aligned} \quad (61)$$

we find $\chi_S^{(6)}$ is not the common eigenstate of $\{\vec{S}^2, s_z\}$, corresponding to $S = 0$ and $\mu = 0$. Their common eigenstate $\chi_S^{(6)}$ can be written as equation (40), we have

$$s_z \chi_S^{(6)} = 0, \quad (62)$$

i.e.,

$$\mu \equiv 0, \quad (63)$$

and

$$\begin{aligned} \vec{S}^2 \chi_S^{(6)} &= 4[a\chi_0(s_{1z})\chi_0(s_{2z}) + b(\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z}))] \\ &\quad + 4b\chi_0(s_{1z})\chi_0(s_{2z}) + 2(a-b)(\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z})) \\ &= 4(a+b)\chi_0(s_{1z})\chi_0(s_{2z}) + 2(a+b)(\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z})) \\ &= 0, \end{aligned} \quad (64)$$

we have

$$\begin{cases} 4(a+b) = 0 \\ 2(a+b) = 0 \end{cases}, \quad (65)$$

i.e.,

$$a = -b, \quad (66)$$

then

$$\chi_S^{(6)} = a\chi_0(s_{1z})\chi_0(s_{2z}) - a[\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z})], \quad (67)$$

by condition of normalization

$$(\chi_S^{(6)})^+ \chi_S^{(6)} = 1, \quad (68)$$

$$a = \frac{1}{\sqrt{3}}, \quad (69)$$

we get the eigenstate $\chi_S^{(6)}$, it is

$$\chi_S^{(6)} = \frac{1}{\sqrt{3}}\chi_0(s_{1z})\chi_0(s_{2z}) - \frac{1}{\sqrt{3}}[\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z})]. \quad (70)$$

(3) $S = 1$ spin wave functions

(a) checking spin wave function $\chi_A^{(1)}$:

$$\begin{aligned} s_z X_A^{(1)} &= (s_{1z} + s_{2z}) \frac{1}{\sqrt{2}} [\chi_0(s_{1z})\chi_1(s_{2z}) - \chi_0(s_{2z})\chi_1(s_{1z})] \\ &= \frac{1}{\sqrt{2}} [(s_{1z}\chi_0(s_{1z}))\chi_1(s_{2z}) - \chi_0(s_{2z})(s_{1z}\chi_1(s_{1z}))] \\ &\quad + \frac{1}{\sqrt{2}} [\chi_0(s_{1z})(s_{2z}\chi_1(s_{2z})) - (s_{2z}\chi_0(s_{2z}))\chi_1(s_{1z})] \\ &= \frac{1}{\sqrt{2}} [\chi_0(s_{1z})\chi_1(s_{2z}) - \chi_0(s_{2z})\chi_1(s_{1z})], \end{aligned} \quad (71)$$

i.e.,

$$\mu = 1, \quad (72)$$

$$\begin{aligned} \vec{S}^2 \chi_A^{(1)} &= 4\chi_A^{(1)} + \sqrt{2}[(s_{1x}\chi_0(s_{1z}))(s_{2x}\chi_1(s_{2z})) - (s_{2x}\chi_0(s_{2z}))(s_{1x}\chi_1(s_{1z}))] \\ &\quad + \sqrt{2}[(s_{1y}\chi_0(s_{1z}))(s_{2y}\chi_1(s_{2z})) - (s_{2y}\chi_0(s_{2z}))(s_{1y}\chi_1(s_{1z}))] \\ &\quad + \sqrt{2}[(s_{1z}\chi_0(s_{1z}))(s_{2z}\chi_1(s_{2z})) - (s_{2z}\chi_0(s_{2z}))(s_{1z}\chi_1(s_{1z}))] \\ &= 4\chi_A^{(1)} + \sqrt{2}[\frac{1}{\sqrt{2}}(\chi_1(s_{1z}) + \chi_{-1}(s_{1z}))\frac{1}{\sqrt{2}}\chi_0(s_{2z}) - \frac{1}{\sqrt{2}}(\chi_1(s_{2z}) + \chi_{-1}(s_{2z}))\frac{1}{\sqrt{2}}\chi_0(s_{1z})] \\ &\quad + \sqrt{2}[-\frac{i}{\sqrt{2}}(\chi_1(s_{1z}) - \chi_{-1}(s_{1z}))\frac{i}{\sqrt{2}}\chi_0(s_{2z}) + \frac{i}{\sqrt{2}}(\chi_1(s_{2z}) - \chi_{-1}(s_{2z}))\frac{i}{\sqrt{2}}\chi_0(s_{1z})] \\ &= 4\chi_A^{(1)} + \sqrt{2}[\chi_1(s_{1z})\chi_0(s_{2z}) - \chi_1(s_{2z})\chi_0(s_{1z})] \\ &= 4\chi_A^{(1)} - 2\chi_A^{(1)} = 2\chi_A^{(1)}, \end{aligned} \quad (73)$$

i.e.,

$$S = 1, \quad (74)$$

we have checked $\chi_A^{(1)}$ is the spin wave function of two-photon corresponding to $S = 1$ and $\mu = 1$.

(b) checking spin wave function $\chi_A^{(2)}$:

$$\begin{aligned}
s_z X_A^{(2)} &= \frac{1}{\sqrt{2}}[(s_{1z}\chi_1(s_{1z}))\chi_{-1}(s_{2z}) - \chi_1(s_{2z})(s_{1z}\chi_{-1}(s_{1z}))] \\
&\quad + \frac{1}{\sqrt{2}}[\chi_1(s_{1z})(s_{2z}\chi_{-1}(s_{2z})) - (s_{2z}\chi_1(s_{2z}))\chi_{-1}(s_{1z})] \\
&= 0,
\end{aligned} \tag{75}$$

i.e.,

$$\mu = 0, \tag{76}$$

$$\begin{aligned}
\vec{S}^2 \chi_A^{(2)} &= 4\chi_A^{(2)} + \sqrt{2}[(s_{1x}\chi_1(s_{1z}))(s_{2x}\chi_{-1}(s_{2z})) - (s_{2x}\chi_1(s_{2z}))(s_{1x}\chi_{-1}(s_{1z}))] \\
&\quad + \sqrt{2}[(s_{1y}\chi_1(s_{1z}))(s_{2y}\chi_{-1}(s_{2z})) - (s_{2y}\chi_1(s_{2z}))(s_{1y}\chi_{-1}(s_{1z}))] \\
&\quad + \sqrt{2}[(s_{1z}\chi_1(s_{1z}))(s_{2z}\chi_{-1}(s_{2z})) - (s_{2z}\chi_1(s_{2z}))(s_{1z}\chi_{-1}(s_{1z}))] \\
&= 4\chi_A^{(2)} + 2 \cdot \frac{1}{\sqrt{2}}\sqrt{2}[\frac{1}{\sqrt{2}}\chi_0(s_{1z})\frac{1}{\sqrt{2}}\chi_0(s_{2z}) - \frac{1}{\sqrt{2}}\chi_0(s_{2z})\frac{1}{\sqrt{2}}\chi_0(s_{1z})] \\
&\quad + \frac{i}{\sqrt{2}}\chi_0(s_{1z})(-\frac{i}{\sqrt{2}})\chi_0(s_{2z}) - \frac{i}{\sqrt{2}}\chi_0(s_{2z})(-\frac{i}{\sqrt{2}})\chi_0(s_{1z}) \\
&\quad - \chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z})] \\
&= 4\chi_A^{(2)} - 2\chi_A^{(2)} = 2\chi_A^{(2)},
\end{aligned} \tag{77}$$

i.e.,

$$S = 1, \tag{78}$$

we have checked $\chi_A^{(2)}$ is the spin wave function of two-photon corresponding to $S = 1$ and $\mu = 0$.

(c) checking spin wave function $\chi_A^{(3)}$:

$$\begin{aligned}
s_z X_A^{(3)} &= \frac{1}{\sqrt{2}}[(s_{1z}\chi_0(s_{1z}))\chi_{-1}(s_{2z}) - \chi_0(s_{2z})(s_{1z}\chi_{-1}(s_{1z}))] \\
&\quad + \frac{1}{\sqrt{2}}[\chi_0(s_{1z})(s_{2z}\chi_{-1}(s_{2z})) - (s_{2z}\chi_0(s_{2z}))\chi_{-1}(s_{1z})] \\
&= -\frac{1}{\sqrt{2}}[\chi_0(s_{1z})\chi_{-1}(s_{2z}) - \chi_0(s_{2z})\chi_{-1}(s_{1z})],
\end{aligned} \tag{79}$$

i.e.,

$$\mu = -1, \tag{80}$$

$$\begin{aligned}
\vec{S}^2 \chi_A^{(3)} &= 4\chi_A^{(3)} + \sqrt{2}[(s_{1x}\chi_0(s_{1z}))(s_{2x}\chi_{-1}(s_{2z})) - (s_{2x}\chi_0(s_{2z}))(s_{1x}\chi_{-1}(s_{1z}))] \\
&+ \sqrt{2}[(s_{1y}\chi_0(s_{1z}))(s_{2y}\chi_{-1}(s_{2z})) - (s_{2y}\chi_0(s_{2z}))(s_{1y}\chi_{-1}(s_{1z}))] \\
&+ \sqrt{2}[(s_{1z}\chi_0(s_{1z}))(s_{2z}\chi_{-1}(s_{2z})) - (s_{2z}\chi_0(s_{2z}))(s_{1z}\chi_{-1}(s_{1z}))] \\
&= 4\chi_A^{(3)} + \sqrt{2}[\frac{1}{\sqrt{2}}(\chi_1(s_{1z}) + \chi_{-1}(s_{1z}))\frac{1}{\sqrt{2}}\chi_0(s_{2z}) - \frac{1}{\sqrt{2}}(\chi_1(s_{2z}) + \chi_{-1}(s_{2z}))\frac{1}{\sqrt{2}}\chi_0(s_{1z})] \\
&+ \sqrt{2}[\frac{-i}{\sqrt{2}}(\chi_1(s_{1z}) - \chi_{-1}(s_{1z}))\frac{-i}{\sqrt{2}}\chi_0(s_{2z}) - \frac{-i}{\sqrt{2}}(\chi_1(s_{2z}) - \chi_{-1}(s_{2z}))\frac{-i}{\sqrt{2}}\chi_0(s_{1z})] \\
&= 4\chi_A^{(3)} - 2\chi_A^{(3)} \\
&= 2\chi_A^{(3)},
\end{aligned} \tag{81}$$

i.e.,

$$S = 1, \tag{82}$$

we have checked $\chi_A^{(3)}$ is the spin wave function of two-photon corresponding to $S = 1$ and $\mu = -1$. By calculation and checking, we obtain the correct spin wave function of two-photon, they are

(1) $S = 2$ spin wave functions

$$\chi_S^{(1)} = \chi_{22}(s_{1z}, s_{2z}) = \chi_1(s_{1z})\chi_1(s_{2z}), \quad (S = 2, \mu = 2) \tag{83}$$

$$\chi_S^{(2)} = \chi_{21}(s_{1z}, s_{2z}) = \frac{1}{\sqrt{2}}[\chi_0(s_{1z})\chi_1(s_{2z}) + \chi_0(s_{2z})\chi_1(s_{1z})], \quad (S = 2, \mu = 1) \tag{84}$$

$$\chi_S^{(3)} = \frac{2}{\sqrt{6}}\chi_0(s_{1z})\chi_0(s_{2z}) + \frac{1}{\sqrt{6}}[\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z})], \quad (S = 2, \mu = 0) \tag{85}$$

$$\chi_S^{(4)} = \chi_{2-1}(s_{1z}, s_{2z}) = \frac{1}{\sqrt{2}}[\chi_0(s_{1z})\chi_{-1}(s_{2z}) + \chi_0(s_{2z})\chi_{-1}(s_{1z})], \quad (S = 2, \mu = -1) \tag{86}$$

$$\chi_S^{(5)} = \chi_{2-2}(s_{1z}, s_{2z}) = \chi_{-1}(s_{1z})\chi_{-1}(s_{2z}), \quad (S = 2, \mu = -2) \tag{87}$$

(2) $S = 0$ spin wave function

$$\chi_S^{(6)} = \frac{1}{\sqrt{3}}\chi_0(s_{1z})\chi_0(s_{2z}) - \frac{1}{\sqrt{3}}[\chi_1(s_{1z})\chi_{-1}(s_{2z}) + \chi_1(s_{2z})\chi_{-1}(s_{1z})], \quad (S = 0, \mu = 0) \tag{88}$$

(3) $S = 1$ spin wave functions

$$\chi_A^{(1)} = \chi_{11}(s_{1z}, s_{2z}) = \frac{1}{\sqrt{2}}[\chi_0(s_{1z})\chi_1(s_{2z}) - \chi_0(s_{2z})\chi_1(s_{1z})], \quad (S = 1, \mu = 1) \tag{89}$$

$$\chi_A^{(2)} = \chi_{10}(s_{1z}, s_{2z}) = \frac{1}{\sqrt{2}}[\chi_1(s_{1z})\chi_{-1}(s_{2z}) - \chi_1(s_{2z})\chi_{-1}(s_{1z})], \quad (S = 1, \mu = 0) \tag{90}$$

$$\chi_A^{(3)} = \chi_{1-1}(s_{1z}, s_{2z}) = \frac{1}{\sqrt{2}}[\chi_0(s_{1z})\chi_{-1}(s_{2z}) - \chi_0(s_{2z})\chi_{-1}(s_{1z})], \quad (S = 1, \mu = -1) \tag{91}$$

From equations (83)-(91), we can find the two-photon spin states of $S = 2$ and $S = 0$ are symmetrical spin states, and the spin states of $S = 1$ are antisymmetrical spin states. In the two-photon

spin states, the states $\chi_S^{(2)}$, $\chi_S^{(3)}$, $\chi_S^{(4)}$, $\chi_S^{(6)}$, $\chi_A^{(1)}$, $\chi_A^{(2)}$ and $\chi_A^{(3)}$ are two-photon entanglement states. We know the two-electron entanglement states are

$$\chi_S = \frac{1}{\sqrt{2}}[\chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z}) + \chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{1z})], \quad (S = 1, \mu = 0) \quad (92)$$

and

$$\chi_A = \frac{1}{\sqrt{2}}[\chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z}) - \chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{1z})], \quad (S = 0, \mu = 0) \quad (93)$$

where $\chi_{\frac{1}{2}}(s_z)$ and $\chi_{-\frac{1}{2}}(s_z)$ are s_z eigenstates of eigenvalues $\frac{1}{2}$ and $-\frac{1}{2}$, i.e., the two-electron entanglement states are expressed by the spin state of two single electron.

At present, the two-photon entanglement states are expressed as [23–26]

$$\chi_1 = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2 \pm |V\rangle_1|V\rangle_2), \quad (94)$$

and

$$\chi_2 = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 \pm |V\rangle_1|H\rangle_2). \quad (95)$$

Where $|H\rangle$ and $|V\rangle$ denote the states of horizontal and vertical linear polarization. We think equations (94) and (95) are classical polarization vector expression, and are not complete. The quantum expression of two-photon spin states should be expressed by the spin state of photon, and the complete entanglement states of two-photon are shown in equations (84), (85), (86), (88), (89), (90) and (91).

3 Conclusion

In a number of literature, the two-photon entanglement states are expressed by equations (94) and (95), they are a classical polarization vector expression and they are not complete. In this paper, we have given the spin states of two-photon, which are expressed by the quadratic combination of two single photon spin states. Obviously, they are a kind of quantum expression. Otherwise, we give all two-photon entanglement states.

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